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# Effect of Complex Phase Plate on Tight Focusing of Azimuthally Polarized Dark and Anti-Dark Gaussian Beam

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## Abstract

*In this work, the focusing of azimuthally polarized Dark and Anti-Dark Gaussian (DADG) beams, through a high numerical-aperture (NA). Lens is investigated throretically by using the vector diffraction theory. It is noted upon suitable phase modulation for the incident DADG beam, the intensity distribution in the focal region can be tuned to be a focal hole segment with multiple focal holes separated with different axial distances. Such a chain of focal hole segment is useful for multiple trapping of low refractive index particle and also an erase beam for STED microscopy with screening rate.*

***Keywords:*** *Flattops; Optical Tubes; Optical bubbles; Three Dimensional Optical tube.*

## 1. INTRODUCTION

Recently, Generation of three-dimensional (3D) optical tube beams that are dark regions in space surrounded by light are driven by wide ranging applications including dark optical traps for atoms and manipulation (Friedman *et al.* 2002), guiding and binding of micro particles and biological cells (Cizmar *et al.* 2010), erase beams for super resolution fluorescence microscopy (Watanabe *et al.* 2003). However, the recent method of generating such an optical tubes suffers remarkable fluctuations due to unexpected diffraction. The DOE(Diffractive optical element) on the pupil plane is also utilized to generate focal structures such as optical tubes, flattops and optical bubbles, useful for optical trapping, optical manipulation and so on (Grimm *et al.* 2000; Tian and Pu, 2011; Lalithambigai *et al.* 2012) In order to accomplish a stable trap of a particle and to move low and high refractive index particles, dissimilar optical gradient forces are necessary and the scattering force considered is always proportional to optical intensity. Three dimensional (3D) multi-site optical trapping requires multiple focal spots for high refractive index particles and multiple focal holes for low refractive index particles in the focal region, which is not simple to attain. A number of methods are reported to construct a multiple optical trap (Lalithambigai *et al.* 2013; Prabakaran *et al.* 2014; Yiqiong Zhao *et al.* 2005; Cao *et al.* 2013), for example recently a phase contrast method to generate an array of focal spots using a multi-beam system is suggested. Recently dark hollow beams (DHBs) with zero central intensity have attracted large interest over the last few years owing to their numerous applications in modern optics, atomic optics and binary optics. Various hollow laser beams with different intensity profiles have been introduced. (Wang *et al.* 2004; Cai and Ge, 2006; Ito *et al.* 1996; Kuga *et al.* 1997; Cai *et al.* 2003; Mei and Zhao, *et al.* 2006a; 2006b; 2008) The dark/antidark beams (DADBs) is one such beam which are partially coherent diffraction-free modes that have been introduced theoretically (Ponomarenko *et al.* 2007) . These beams can have intensity profiles with dips (or peaks) on a background, so they can serve as atomic traps. Zhu etal.first suggested the method of superposition of uncorrelated Bessel modes by means of holographic technic to generate DADBs (Zhu *et al.* 2019). Hyden etal.by using the genuine cross-spectral density function criterion (Hyde and Avramov-Zumarovic, 2019), a fully coherent class of dark/antidark paraxial modes named dark/antidark Bessel–Gaussian beams (DADGBs) has been proposed (Faroq and Belafhal, 2018). The propagation characteristics of these beams in a turbulent

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atmosphere have also been examined more recently (Yaalou *et al.* 2019).The objective of the study reported in this work is to investigate the effect of complex phase filter (CPF) in the focal properties of the tightly focused azimuthally polarized dark and anti-dark Gaussian beam. We observed that by properly designing suitable complex phase filter for the incident DADG beam, one can achieve many novel focal patterns including splitting of focal rings and generation of multiple focal hole structures.

## 2. THEORY

The analysis was performed on the basis of Richards and Wolf's vectorial diffraction method widely used for high NA focusing systems at arbitrary incident polarization. The electric field distribution near in the focusing for the incident azimuthally polarized beam is given by (Rucgards abd Wolf, 1959).

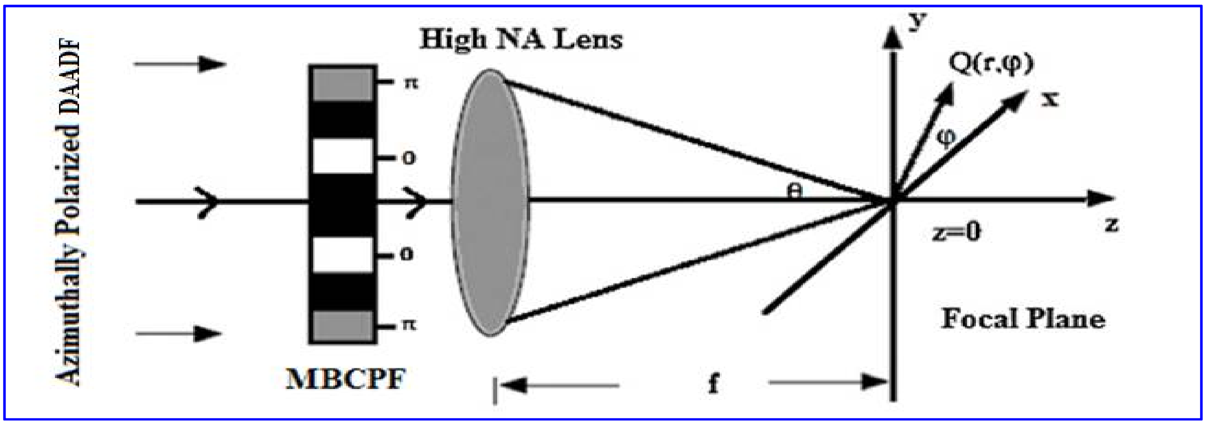


**Fig. 1: Tight focusing of azimuthally polarized beam dark and anti-dark beam passes through a complex phase filter and is subsequently focused by a high-NA lens.**

$$E(r,\varphi,z)=\begin{bmatrix}E_r\\E_\varphi\\E_z\end{bmatrix}=\left[2A\int_0^{\alpha}\cos^{1/2}(\theta)\sin(\theta)\,A(\theta)\,J_1(kr\sin\theta)\,e^{ikz\cos\theta}\right]d\theta\rightarrow(1)$$

where A is relative amplitude, $\theta_{max}$ = *arcsin (NA/n)* that is the maximum aperture angle with (NA/n) is the ratio of numerical aperture (NA) and n is the index of refraction between the lens and the sample.*k*is the wave number in free space. $J_n(\theta)$ denotes the nth-order Bessel function and the function of $T(\theta)$ describe the amplitude modulation.The analytical expression of the incident DADG beams in source plane z=0 is given by (Yaalou *et al.* 2019)

$$E_0(r,z=0)=(1+\alpha\,J_0(2\beta r))\exp\left(\frac{-r^2}{w_0^2}\right)\rightarrow(2)$$

Where $J_0$ is thezeroth order Bessel function of the first kind, $\alpha$ and $\beta$ are arbitrary constants, $\beta$ being real value and $w_0$ is the width of the Gaussian part. Obviously, for $\alpha=0$, the beam, governed by Eq. (2.) reduces a conventional Gaussian beam. On other hand, for $\alpha=$-1 and 1, under these conditions, the incident beam field, established by Eq. (2.), reduces to ADG and DG beams, respectively.For the objective that obeysthe sine condition (Lin *et al.* 2011; Dehez *et al.* 2012), Eq. (2.) stands to

$$E_0(\theta)=\left(1+\alpha J_0\left(\frac{2\mu\sin\theta}{NA}\right)\right)\exp\left[-\frac{\sin^2\theta}{NA^2.w^2}\right]\rightarrow(3)$$

Where $w=w_0/\rho_0$ called the relative waist width, $\mu=\beta\rho_0$ is the ratio of the pupil radius, $NA=\rho_0/f$ and $E_0(\theta)$ represents the amplitude variation of the DADG beam. The dedicated complex phase filter is shown in Fig. 1. The CPF is the combination of phase and amplitude filter and it consists 4 belts in the radial direction. Multi-zone phase and amplitude filters are the most useful DOEs to decrease the focal spot size and to improve the focal depth (Xu *et al.* 2007). The effect multi belt complex phase filter on the input azimuthally polarized dark and anti-dark beam is evaluated by $T(\theta)$

$$T(\theta) = \begin{cases} 0, \ for 0 < \theta < \theta_1, \theta_2 < \theta < \theta_3, \\ 1, \ for \theta_1 < \theta < \theta_2, \\ -1 \ for \theta_3 < \theta < \theta_{\max} \end{cases} \rightarrow (4)$$

Here we considered that the set of four angles to obtain particular focal patterns are optimized by traditional global-search-optimization algorithm. Based on this algorithm, we choose one structure with random values for $\theta_1$ to $\theta_3$ from all possibilities and simulate their focusing properties by vector diffraction theory. If the structure generates a sub wavelength single focal spot and satisfies the limiting conditions that that the Full Width Half Maximum (FWHM) of the generated focal spot segment is less than 0.5λ, it is chosen as the initial structure during the optimization procedures. In the following steps, we continue to vary $\theta$ of one chosen zone to generate multiple focal spots segments by considering the limiting conditions that that the FWHM of each of the generated focal spot is less than 0.5λ and there should be at least five, three or two such focal spots in the focal segment.

## 3. RESULTS & DISCUSSION

We perform the integration of Eq. (1) numerically using parameters λ = 1, and NA of the objective is 0.9. Here, for simplicity, we assume that the refractive index n = 1 and A = 1, For all calculation in the length unit is normalized to λ and the energy density is normalized to unity. Fig. 2. illustrates the evolution of three dimensional light intensity distribution of the high NA lens for the incident phase modulated dark hollow beam. The set of angles of complex phase filter optimized for the generation of multiple focal holes are $\theta_1=50.90^0$, $\theta_2=54.85^0$ *and* $\theta_3= 60.83^0$. $\theta_{max}= 64.19^0$.

Fig. 2.(a) illustrates the splitting of focal hole segment generated by the high NA lens for the phase modulated dark beam. It is observed from Fig. 2.(a) that the generated focal hole segment are uniformly separated in the focal region and each focal hole has the focal depth around 2.8λ measured at r = 0.3λ and the axial distance of separation of each focal hole is 1.8λ and is shown in Fig 2.(c). It is observed from the fig. 2.(b) that the FWHM of each ring is 0.37λ.

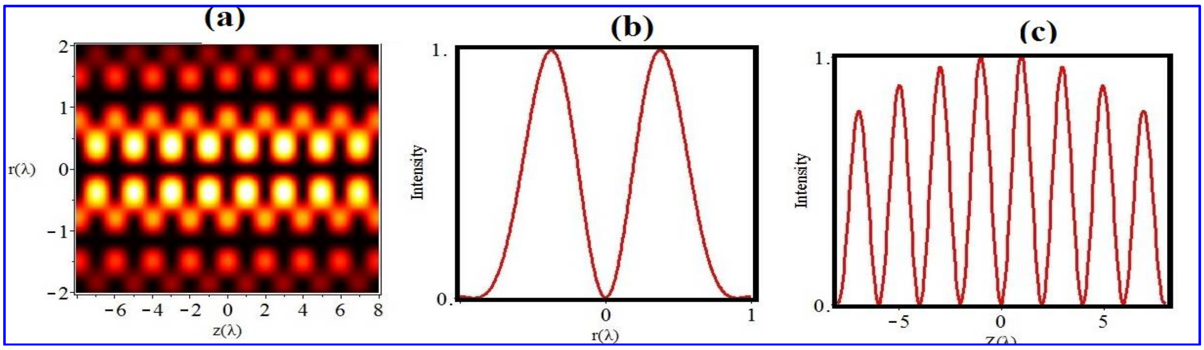


**Fig. 2: (a) total intensity distribution in the r–z plane for phase modulated azimuthally polarized Dark hollow Gaussian beam(α=1) (b) Intensity distributions of the radial direction at r = 0.35λ. c) Axial intensity distribution z=1.1λ.**

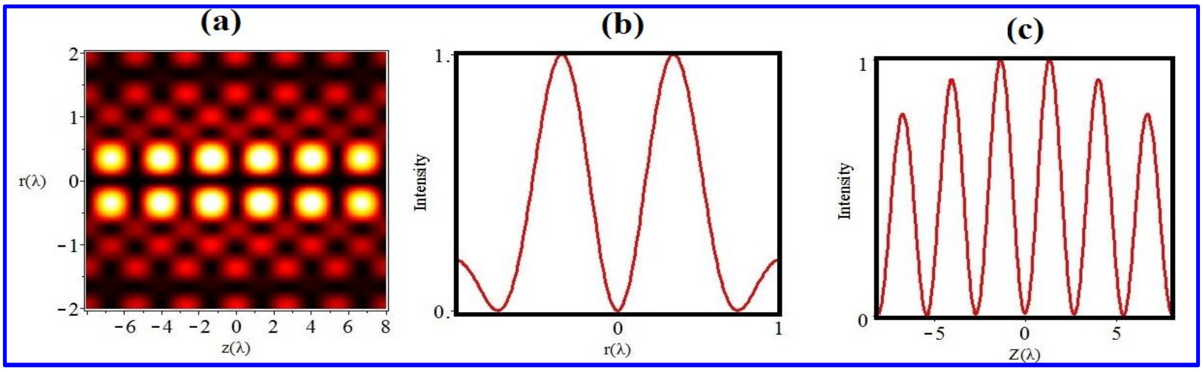


**Fig. 3: (a) total intensity distribution in the r–z plane; (b) Intensity distributions of the radial direction at r = 0.35λ. c) axial intensity distribution z=1.1λ.**

Fig. 3.(a) describes the array of focal hole segment generated by the high NA lens for the CPF with angles optimized as $\theta_1$=50.90$^0$, $\theta_2$= 56.85$^0$and $\theta_3$= 60.83$^0$. θ max= 64.19$^0$ . From the fig.3.(a-b) we observed that the focal holes segment contains 6 focal holes of almost uniform intensity, each having FWHM of 0.362λ and are axially separated by the distance of 1.6λ . The on axial intensity of the generated focal hole segment calculated at r = 0.3λ is shown in Fig. 8. 4(c) and it is observed from the fig.3.(c) the DOF of each focal hole as 2λ .

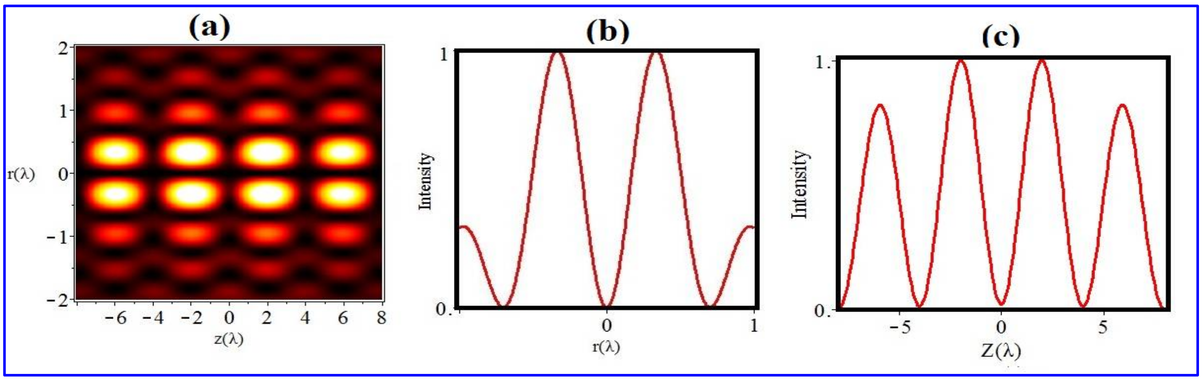


**Fig. 4: (a) total intensity distribution in the r–z plane; (b) Intensity distributions of the radial direction at r = 0.35λ. c) Axial intensity distribution z=1.1λ.**

Fig. 4(a-c). shows the generation of four focal hole pattern by CPP optimized with angles θ1=50.90$^{\circ}$, θ2= 56.85$^{\circ}$ and θ3= 60.83$^{\circ}$, $\theta_{max}$= 64.19$^{\circ}$. From the fig. we observed that the generated focal holes segment contains four focal holes and the light intensities of these four focal holes are approximately equal and each having FWHM of 0.345λ and are separated by axial distance of 2λ between them. It is observed that each focal hole having focal depth around 2.8λ as measured from intensity distribution at r = 0.3λ and is shown in Fig. 5(c).

Fig. 5(a) shows the total tow dimensional intensity distribution of the axially extended focal hole generated by the proposed high NA lens with dedicated CPF. It is observed from the fig.8.5(c) that the generated focal hole extends up to 3λ with uniform axial intensity. From fig. 5(b) the FWHM of the generated focal hole is measured as 0.332λ. The set of three angles of the CPP optimized to achieve the above mentioned focal hole segment are $\theta_1$=50.90$^{\circ}$, $\theta_2$= 56.85$^{\circ}$and $\theta_3$= 58.83$^{\circ}$. $\theta_{max}$ = 64.19$^{\circ}$ Table 1 shows the angle of the CPP optimized for the generation of focal hole segment for incident azimuthally polarized Dark and Anti-Dark Hollow Gaussian beam. Such a sub wavelength scale long focal depth may find applications in optical, biologicaland atmospheric sciences.

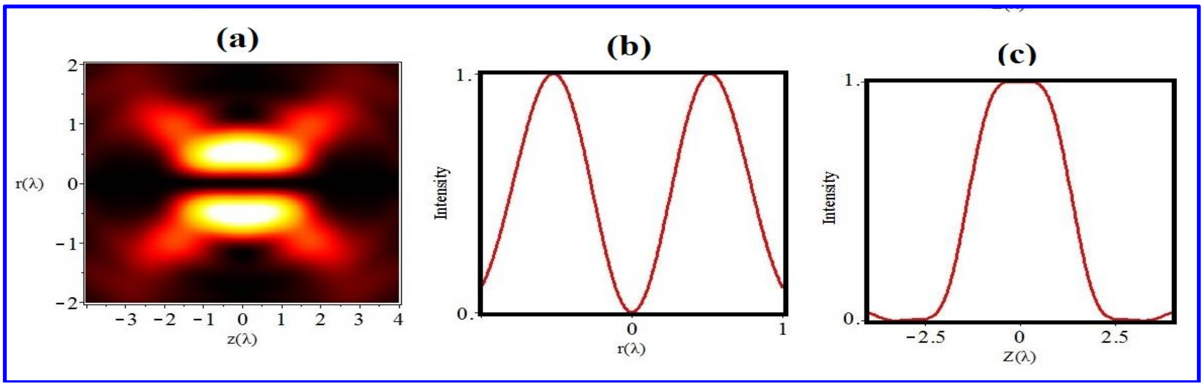


**Fig. 5: (a) total intensity distribution in the r–z plane for phase modulated azimuthally polarized Anti-Dark hollow Gaussian beam(α= -1); (b) Intensity distributions of the radial direction at r = 0.35λ. c) Axial intensity distribution z=1.1λ.**

**Table 1. FWHM and DOF values of Focal holes generated for different beam parameter α and complex phase filter**

| S.No | DADG beam parameter | Optimized angle of CPF | Number of Focal holes | FWHM (λ) | DOF (λ) | Axial separation (λ) |
|---|---|---|---|---|---|---|
| 1 | +1 | $\theta_1 = 50.90^\circ$, $\theta_2 = 54.85^\circ$ and $\theta_3 = 60.83^\circ$ $\theta_{max} = 64.19^\circ$ | 8 | 0.37 | 2.8 | 1.8 |
| 2 | +1 | $\theta_1 = 50.90^\circ$, $\theta_2 = 56.85^\circ$ and $\theta_3 = 58.83^\circ$ $\theta_{max} = 64.19^\circ$ | 6 | 0.36 | 2 | 1.6 |
| 3 | +1 | $\theta_1 = 46.90^\circ$, $\theta_2 = 56.85^\circ$ and $\theta_3 = 60.83^\circ$. $\theta_{max} = 64.19^0$ | 4 | 0.34 | 2.8 | 2 |
| 4 | -1 | $\theta_1 = 50.90^\circ$, $\theta_2 = 56.85^\circ$ and $\theta_3 = 60.83^\circ$. $\theta_{max} = 64.19^\circ$ | 1 | 0.33 | 3 | Uniform axial intensity |

## 4. CONCLUSION

In conclusion, The focusing of azimuthally polarized Dark and anti-dark Gaussian (DADG) beams, propagating through a high numerical-aperture (NA) Lens is investigated theoretically by using the vector diffraction theory, It is possible to generate many novel focal patterns such as focal hole segments contains different number of focal holes with uniform intensity distribution and axially extended sub wavelength focal hole etc. We also observed that the number of focal hole and axial distance between them can be tuned by effectively tuning the phase of the incident azimuthally polarized Dark and anti-dark Gaussian (DADG) beams. Such a tunable multiple focal hole with variable axial distance is an effective tool for optical trapping for low refractive index particles can be achieved precisely and controllably.

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